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Axial directions of folds in rocks with linear/planar fabrics

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Abstract—This paper examines the geological implications of an analysis of constraints on the orientation of fold axes in orthotropic materials. We argue that rocks with penetrative linear and planar shape fabrics may have orthotropic (anisotropic) properties during deformation. Two forms of anisotropy (rheological and structural) could be potentially important in the control of fold axial directions. We discuss a model of deformation of rocks with linear/planar fabrics where, in a single deformation event, major fold axes need not be parallel to minor fold axes and neither need be perpendicular to the principal compressive stress direction. Geological and model examples of anistropic control on fold axial directions are given.

INTRODUCTION

A STRIKING feature of many orogenic belts is that later folds are often symmetrically orientated with respect to earlier formed fabrics. Early shape fabrics in many areas have linear elements and later folds have axes parallel to them.

For example, at Ben Hutig, Sutherland, Scotland, axes of crenulation hinges are parallel to quartz rods (Wilson 1953). Late fold hinges in the deformed conglomerates at Bygdin, Norway and Lebendun, Switzerland are parallel to the long axes of deformed pebbles (Hossack 1968, Aleffi pers. comm. 1980). Also, successive phases of folding in many orogenic zones are frequently coaxial.

One obvious interpretation of this symmetry is that the principal axes of stress and strain during the deformation history remain constant in direction but change in relative magnitude (Talbot 1975). Alternatively, the possibility exists that the early tectonite fabric exerts an active anisotropic control on later deformation and so controls the attitude of the later folds (e.g., Flinn 1962, pp. 426–427).

This paper investigates the latter hypothesis and further examines the geological implications of an analysis of constraints on the orientation of fold axes in orthotropic materials (Cobbold & Watkinson 1981). We discuss a model of deformation of rocks with linear/planar fabrics in order to emphasize the effect of scale. In a single deformation event, major fold axes need not be parallel to minor fold axes and neither need be perpendicular to the principal compressive stress direction. Geological and model examples of anisotropic control on fold axial directions are given.

RELATIONSHIP BETWEEN FABRIC AND ANISTROPY

For a statistically homogeneous rock (Paterson & Weiss 1961), the rheological behaviour can be expressed in terms of bulk or averaged properties. Well-foliated rocks with planar tectonite fabrics exhibit fold structures such as kinks and chevron folds that have been interpreted in terms of theories of deformation of a homogeneous anisotropic medium (Cobbold *et al.* 1971). From comparison of natural structures with theoretical work (Bayly 1964, Biot, 1965, Cobbold 1976), model experiments (Bayly 1969, Cobbold *et al.* 1971, Latham 1979) and experimental rock deformation (Paterson & Weiss 1966), it appears that rocks with planar fabrics have potentially anisotropic rheological properties.

Therefore, it seems likely that rocks with linear/planar fabrics will also possess a linear element of rheological anisotropy which will be another important component in determining mechanical behaviour (Watkinson & Cobbold 1973). Certainly a linear component exists in the elastic properties of metamorphic rocks with tectonite LS or L fabrics (Johnson & Wenk 1974).

Examples of elements which make up a linear/planar fabric are:

(A) microfabrics – acicular (e.g., amphiboles) and platy (e.g., micas) minerals;
(B) deformed inclusions such as pebbles, fossils, and oolites;
(C) structures derived from layers, such as pinch and swell axes, boudin axes and fold hinges and
(D) intersecting foliations or "s"-surfaces such as intersecting shear zones, bedding/cleavage intersection or cleavage/cleavage intersections. These may impart a linear element of anisotropy.

Depending on conditions, for example, of temperature, differential stress and, therefore, deformation mechanism, any one of these elements may be the predominating contributor towards the rheological anisotropy of the rock. Following Neumanns' principle (Paterson & Weiss 1961) we expect the symmetry axes of rheology to coincide with the symmetry axes of the distribution of structural elements.

RHEOLOGICAL AND STRUCTURAL ANISOTROPY

For infinitesimal deformations elastic and linear viscous behaviours are formally analagous. Although most rocks probably deform according to more complex laws we assume that the linear models are sufficient to illustrate the mechanical principles involved.

If rock with a linear/planar fabric has orthotropic properties, it will have rheological properties that are symmetric with respect to three orthogonal planes (Biot 1965, p. 82). For a specially orthotropic material, with linear elastic behaviour, nine constants are required to define the relationship between stress and strain:

$$\begin{aligned} e_{11} &= s_{1111}\,\sigma_{11} + s_{1122}\,\sigma_{22} + s_{1133}\,\sigma_{33}, \\ e_{22} &= s_{2211}\,\sigma_{11} + s_{2222}\,\sigma_{22} + s_{2233}\,\sigma_{33}, \\ e_{33} &= s_{3311}\,\sigma_{11} + s_{3322}\,\sigma_{22} + s_{3333}\,\sigma_{33}, \\ e_{12} &= 2s_{1212}\,\sigma_{12}, \\ e_{23} &= 2s_{2323}\,\sigma_{23}, \\ e_{13} &= 2s_{1313}\,\sigma_{13}, \end{aligned} \tag{1}$$

(Cobbold & Watkinson, eq. 3(a)–(f), 1981).

where the s_{ijkl} are constants and components of an elastic compliance tensor.

When such a material is deformed the axes of strain and, therefore, the potential fold axes need not be parallel to the axes of stress. For example, for cylindrical bending of an orthotropic plate, where compression is applied parallel to the layer anistropy within which there is a linear anisotropy, the relationship between principal directions of stress, α, and strain, β, is given by:

$$\tan 2\alpha = \sin 2\beta/(A\cos^2\beta - B\sin^2\beta), \tag{2}$$

(Cobbold & Watkinson, eq. 14, 1981).

where:

$$A = E_1(1 - \nu_2)/2G(1 - \nu_1\nu_2),$$
$$B = E_2(1 - \nu_1)/2G(1 - \nu_1\nu_2),$$

with Young's modulus E, Poisson's ratio ν, and shear modulus G. In terms of the constants in equation (1):

$$s_{1111} = 1/E_1$$
$$s_{2222} = 1/E_2$$
$$s_{1122} = -\nu_2/E_2$$
$$s_{2211} = -\nu_1/E_1$$
$$s_{1212} = 1/4G.$$

These equations describe the rheological response of the orthotropic material.

There is, however, another form of anisotropic behaviour which may have a significant control on fold axis directions in deformed orthotropic rocks. To demonstrate, let us consider the simple first order bending equations for an elastic plate.

For small slopes of the neutral plane ($\leq 20°$), the bending moment, M of an elastic plate is given by:

$$M = \frac{EI}{1 - \nu^2} \cdot \frac{d^2 v}{dx^2}$$

(e.g., Johnson 1970, p. 217)

where E is Young's elastic modulus, ν Poisson's ratio, I the moment of inertia of the cross-section of a plate with respect to its neutral plane, v the deflection normal to the coordinate axis x. For a unit width,

$$I = \frac{t^3}{12},$$

where t is the thickness of the plate (Fig. 1a).

The bending moment is dependent both on the elastic modulus, E, and the moment of inertia, I of the cross-section. If the plate is isotropic, the modulus E will be of equal value in all directions. However, for an orthotropic plate, the coefficients of elastic compliance will vary in the plane of the plate, resulting in the rheological anisotropy. For example, for a rotation ϕ within the plane of the plate, the principal elastic compliance, s_{1111}, will vary (Fig. 1b) by:

$$s^R_{1111} = s_{1111}\cos^4\phi + (2\,s_{1122} + 4\,s_{1212})\cos^2\phi\sin^2\phi + s_{2222}\sin^4\phi \tag{3}$$

or, in terms of Young's modulus E, Poisson's ratio ν and shear modulus G, as follows:

$$s_{1111} = \frac{1}{E_1},\ s_{2222} = \frac{1}{E_2},$$
$$s_{1122} = -\nu_2/E_2 = s_{2211} = -\nu_1/E_1,$$
$$s_{1212} = \frac{1}{4G}.$$

Then,

$$E^R = E_1/\cos^4\phi + \left(\frac{E_1}{G} - 2\nu_1\right)\sin^2\phi\cos^2\phi + \frac{E_1}{E_2}\sin^4\phi \tag{4}$$

(Lekhnitskii 1968, p. 47).

Therefore, depending on the relative values of E_1, etc. there will be a marked anisotropy of bending or flexural rigidity within the plane of the plate.

The moment of inertia I could also vary. This is primarily a function of the thickness and shape of the plate ($I = t^3/12$). For example, a pinch-and-swell vein will have a variable thickness, t, and, therefore, a variable moment of inertia leading to what is termed by engineers a structural anisotropy. A corrugated or folded vein will also have a variable bending resistance due to structural anistropy (Troitsky 1976) (Appendix). This form of anisotropy will occur even if the material is rheologically isotropic. A good example of the importance of the control of a structural anisotropy is one of the model experiments carried out by Ghosh & Ramberg (1968, plate IV). A set of folds formed during one deformation were re-deformed, the compression direction for the second deformation being at an angle of about thirty degrees to that for the first deformation. No new system of folds

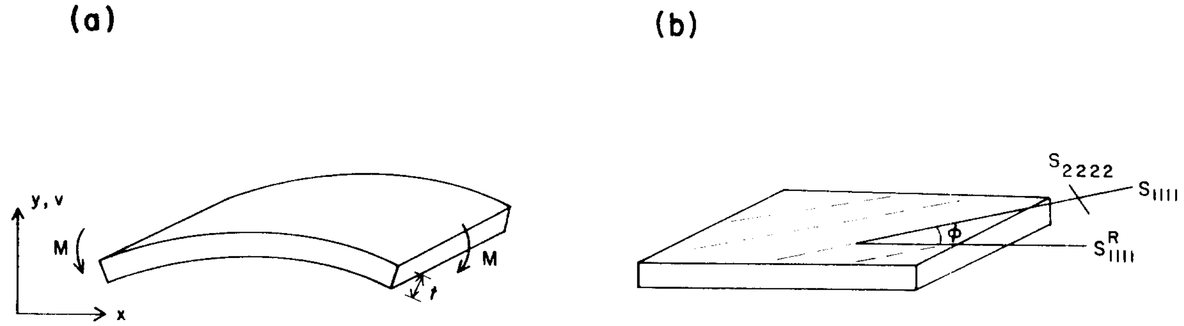

Fig. 1a. Bending of an elastic plate, thickness t, by a bending moment M. The deflection, v, is normal to the coordinate axis x.

Fig. 1b. s^R_{1111} is the modulus in the direction at an angle ϕ to the principal modulus s_{1111} within the plane of the plate.

developed during the second deformation. Instead, the earlier folds were simply tightened and rotated. The structural anisotropy formed by the first deformation was sufficient to control the second. A good geological example is provided by Goguel (1962, fig. 123, p. 168) who states "In a slightly folded region a deformation by accentuation of old folds is quite easy, but the development of folds of different directions is very difficult. Consequently, there will be a tendency to folding along old folds . . .". He gives an example of the reactivation of early folding in the Baronnies region in the sub-alpine folds of Provence, S. W. France.

In general, the bulk anisotropic properties of the rock will be made up of both structural and rheological anisotropic elements. A complex pattern of fold axial directions could develop in deformed rocks with linear/planar fabrics as a result of the control of these two forms of anisotropy (rheological and structural). To demonstrate the potential complexity, we will discuss a hypothetical model which is based on several field observations.

An orthotropic system (Fig. 2a) is folded by a later deformation, the linear element of the orthotropic system being at an angle ϕ to the principal compressive stress. The direction of axes of folds forming on the scale of the whole system is controlled by the direction of the principal strain axes, related to stress direction by an equation such as (2). Within the system there are isolated, single-layer pinch-and-swell veins with an isotropic microfabric which are folded by this later deformation. The minor scale folds are controlled principally by the structural anistropy of the pinch-and-swell veins and form nearly parallel to their axes (Fig. 2b). Thus, in the hypothetical model the minor fold axes form at an angle to the major fold axes and neither are at ninety degrees to the principal compressive stress.

Figure 3 shows a model experiment where minor folds formed at an angle to the major fold, both fold sets forming in the same deformation event. A single layer of competent modelling clay was embedded in a softer clay matrix and compressed in plane strain in a deformation box. The competent layer was initially planar and reinforced by wax rods at an angle of 65° to the principal direction of shortening. The major folds axis, F, formed perpendicular to the maximum shortening direction (in this model there was no rheological anistropy), whereas the minor folds, f, formed almost parallel to the rods. Because of the basic strain incompatibility caused by different folding directions forming at the same time, shearing as local décollement or parallel to the early linear elements may arise (Goguel 1962, p. 169, Watkinson in preparation).

GEOLOGICAL EXAMPLE OF ANISOTROPIC CONTROL ON FOLD AXIAL DIRECTIONS

It is a difficult problem to find geological examples where it is unambiguously clear that anisotropic control has taken place. We have looked for areas where distinctly anomalous trends of late fold structures are related to a distinct local development of early fabrics. One such example occurs in Silurian shales of the Arra Mountains (SW Ireland) which are affected by large folds of variable plunge. Calcite plates with a fibre growth occur in zones along bedding planes, presumably indicating bedding-plane slip during the initial stages of folding. It appears that as the folds developed further, the flexural slip component diminished and further shortening across the folds was expressed as crenulations of the calcite layers. The crenulation fold axes are grossly parallel to the axes of the major folds and to the bedding/cleavage intersection. We will call this direction F.

The angle ϕ between F and the fibres varies between 90° and 12°. For $30° < \phi < 90°$, the fibres are predominantly folded about F; but for $\phi < 30°$, wherever the fibres appear to be strongly developed (in zones, see Fig. 4), the crenulation axes switch parallel to the early fibre direction rather than fold the lineation at a low angle to F (12°–30°). This example seems convincing because it is only in zones where the linear fibres are well developed that the crenulation fold axes switch parallel to them.

CONCLUSIONS

The analysis of the rheological behaviour of ortho-

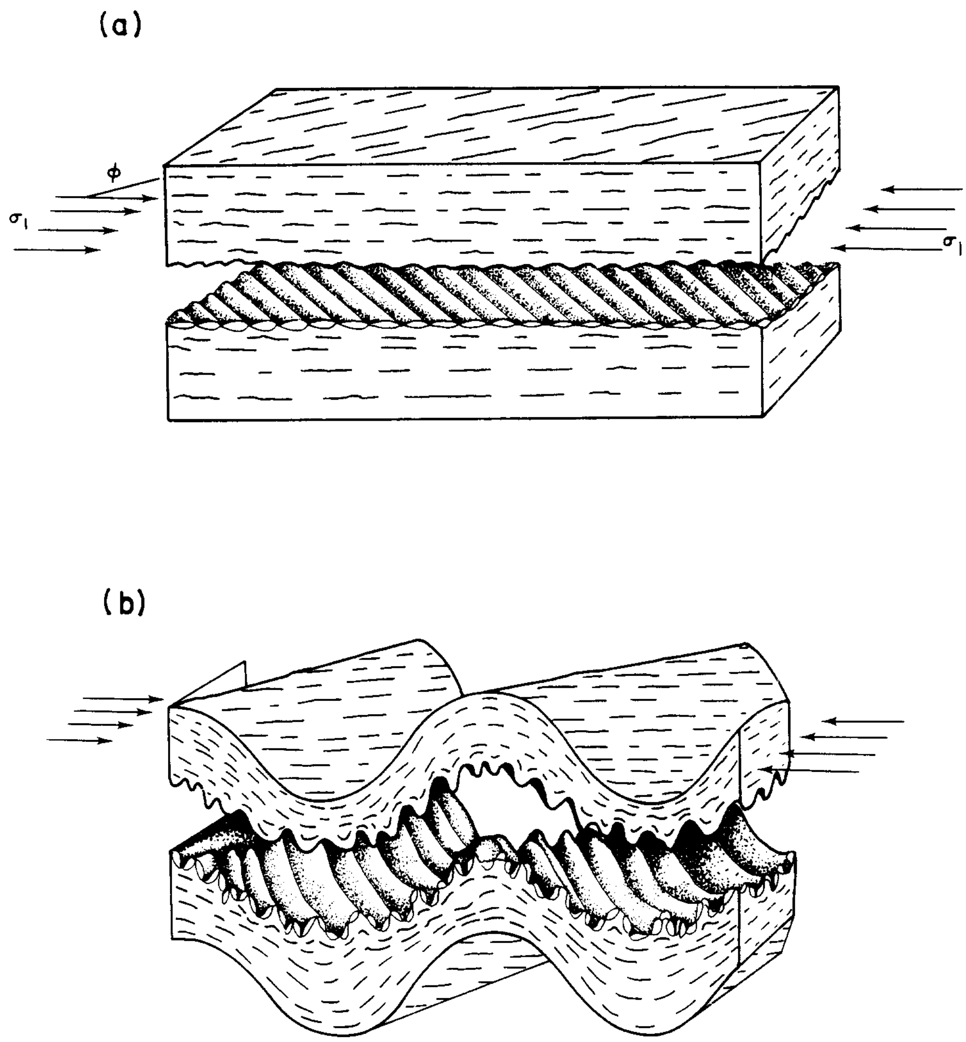

Fig. 2a. A hypothetical model of a rock with orthotropic properties and with a single-layer pinch and swell vein. The lineation is at an angle ϕ to the principal compressive stress.

Fig. 2b. The major folds form at an angle to the lineation and are non-perpendicular to the compressive stress. The minor folds, folding the pinch-and-swell structure, are controlled by the structural anisotropy of the pinch-and-swell vein and form almost parallel to the pinch and swell axes.

tropic materials shows clearly that in general the direction of strain will not be parallel to the direction of applied stress. Potentially then, the direction of fold axes in deformed rocks with orthotropic properties will be influenced by the anisotropy produced by an early penetrative deformation. Two forms of anisotropy (rheological and structural) could be potentially important in the control of fold axial directions.

An appreciation of the potential mechanical controls would seem to be important for the problem of correlating fold phases from area to area. Indeed the anisotropic control of earlier fabrics on later folding may well be the mechanical explanation of why later fold phases, as observed by, for example, Mukhopadhyay (1965) and Tobisch (1967) in the Scottish Caledonides, are so divergent in axial directions.

The crenulation folds in the Arra Mountains appear to be a vivid example of anisotropic control. In other areas, such as the Isle de Groix, Brittany (Quinquis 1980), where crenulation folds follow the variable trend of an earlier penetrative lineation, anisotropic control would seem to be a reasonable explanation of the pattern of deformation.

Until we have more data on values of ductile anisotropy it is difficult to assess the significance of the symmetry of deformation observed in many other areas. Certainly, if high values of anisotropy exist, the symmetry could well be the result of anisotropic control by an early penetrative fabric.

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Fig. 3. Plan and profile view of a layer containing linear rods (f) at an angle to the major fold axis (F). The minor folds form parallel to the rods and contemporaneously with the major fold, but are oblique to the major fold axis F.

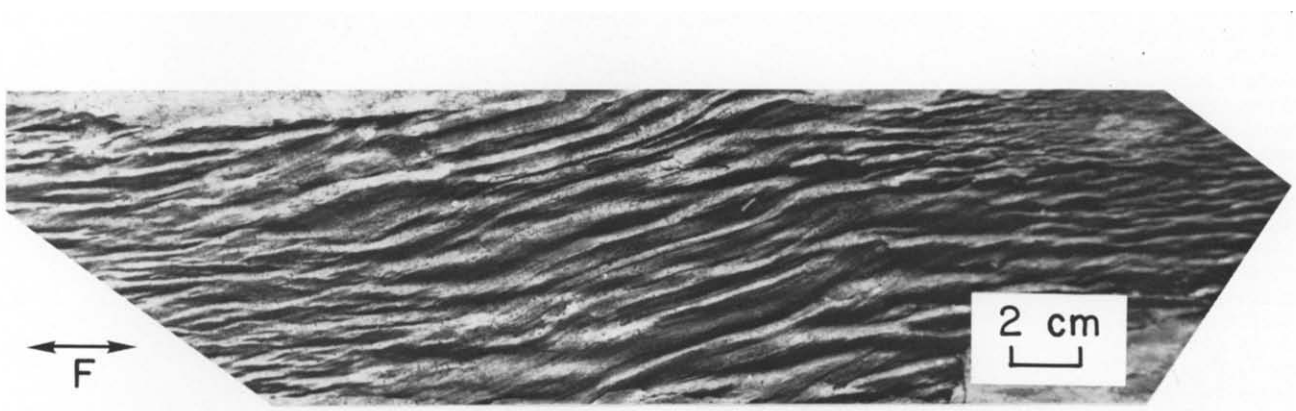

Fig. 4a. Crenulated calcite plates with a fibre growth. Where the fibre growth is well developed, the crenulation fold axes switch parallel to the fibres. Arra Mountains, S. W. Ireland.

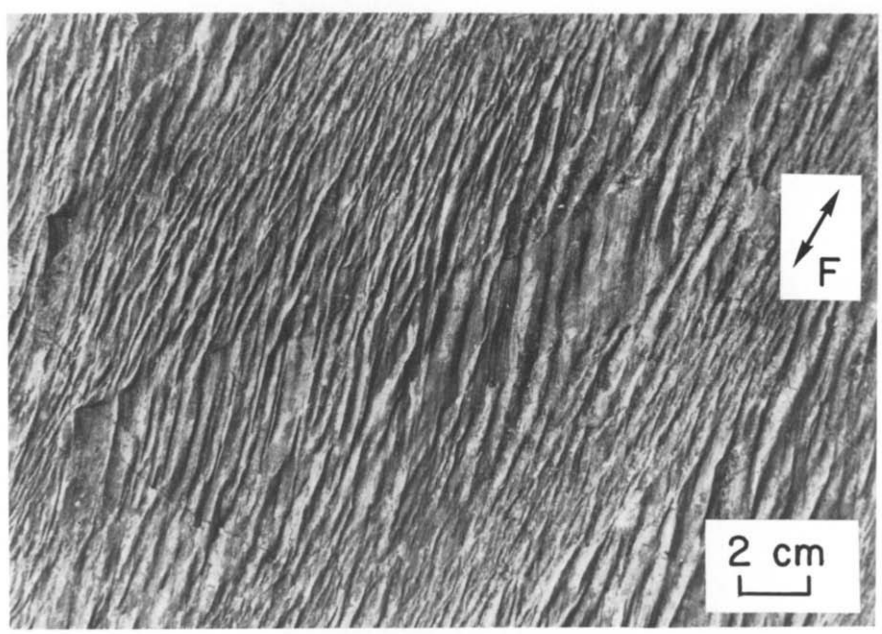

Fig. 4b. Zones of well developed fibre growth between zones where many of the crenulation axes are parallel to the axes of the major folds (direction *F*). Arra Mountains, S. W. Ireland.

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APPENDIX: Structural anisotropy of a corrugated plate

Troitsky (1968) gives an example of the structural anisotropy of a corrugated elastic plate. The plate is unconfined and of isotropic material having corrugation in the x direction (Fig. 5).

For a sinusoidal corrugation of the form

$$y = a \sin \frac{\pi x}{l},$$

the approximate formula for the flexural rigidity (D) in the x direction is

$$D_x = \frac{l}{s} \frac{Et^3}{12(1-v^2)}$$

In the z direction,

$$D_z = EI_z$$

for corrugations with a chord length of one semi-wavelength l, arc length s, thickness t, Young's modulus E, Poisson's ratio v and I_2 is the mean moment of inertia in the xz plane per unit length equal to:

$$I_z = 0.5\,a^2 t \left[1 - \frac{0.81}{1 + 2.5\left(\frac{a}{2l}\right)^2} \cdot \right]$$

(Troitsky 1976, p. 81).

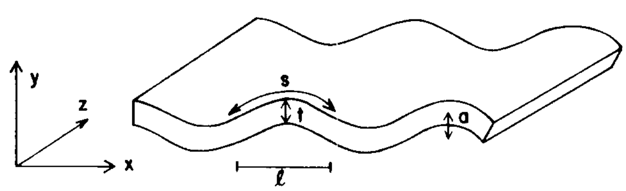

Fig. 5. A corrugated plate of thickness t with a chord length of one semiwavelength l and arc length s.